# Statistical Reasoning

"The most important questions of life are, for the most part, really only problems of probability." — Pierre-Simon, Marquis de Laplace



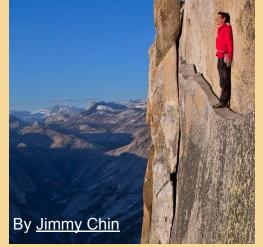






Does the unprescribed use of Ritalin (yes, no) lead to mental illnesses, such as sleeplessness and psychological dependence (yes, no)?





How does the amount of daily physical activity (measured in hours) predict the likelihood of experiencing anxiety symptoms (yes, no)?

Is there a difference in average test scores among students from different teaching methods (traditional, online, hybrid)?





What is the relationship between hours of sleep per night and cognitive function (measured by memory recall)?

# What are you supposed to know already?







CANVAS

BY INSTRUCTUR









#### Methoden van Onderzoek en Statistiek

Samples & sampling distributions

**NHST** 

P values

Confidence intervals

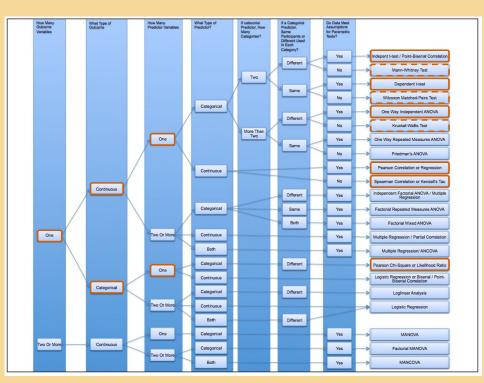
**Probabilities** 

Bayes theorem

Test choice & research design

Data visualization

# What are you supposed to know already?



#### Methoden van Onderzoek en Statistiek

Binomial test

Chi-squared test

Student's *t*-test

One-way ANOVA

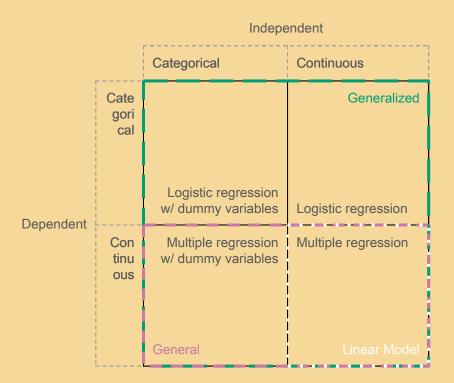
Correlation

Simple linear regression

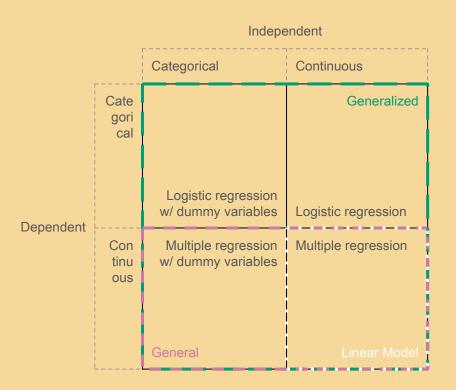
Some nonparametric alternatives

		Indepe	pendent	
		Categorical	Continuous	
Dependent	Cate gori cal			
		Chi-squared test		
	Con tinu ous	Student's <i>t</i> test One-way ANOVA Factorial ANOVA	Simple regression Multiple regression	

☐ From one predictor to multiple predictors



- ☐ From one predictor to multiple predictors
- From a toolbox to a multitool ("Swiss Army knife")



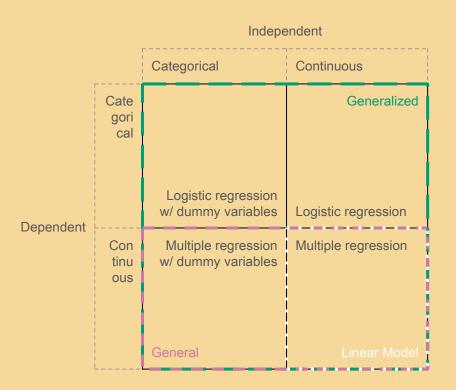
- ☐ From one predictor to multiple predictors
- ☐ From a toolbox to a multitool
- ☐ Hierarchical data analysis

### Independent data



#### Dependent data





- ☐ From one predictor to multiple predictors
- ☐ From a toolbox to a multitool
- ☐ Hierarchical data analysis
- Bayesian inference (vs. frequentist inference)





R Studio





Knowledge and skills in statistical reasoning and inference.

- Conceptual understanding of statistical inference (frequentist, Bayesian and non-parametric statistics)
- Match statistical test to research design (and vice versa)
- Perform statistical tests with R
- Interpret test results from R

Foundation for follow-up courses (e.g., *Experimentation Year 2*) and independent learning.



USINGR

Created with R package ggfx

# Why should you (not) trust me?

#### Me

- Assistant Professor in Psychological Methods
- Not a statistician (but well-connected)

#### Book

- ☐ Written by experts (see "About the Authors")
- Recently written (pros and cons)
- □ Supported by evidence (see "Bibliography")
- ☐ Well-reviewed by experts (see <u>reviews</u>)

## Purpose

- See "How can you benefit…"
- No hidden agenda

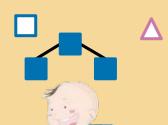


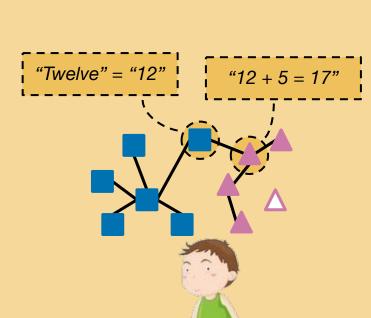
By Frederick Burr Opper (1894)

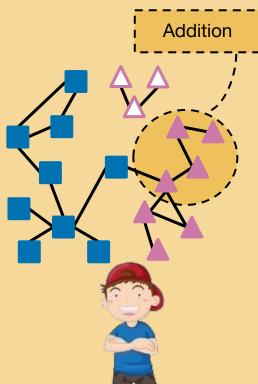
# The wiring of intelligence

Savi et al., 2019

- Incorrect
- Correct
- Language
- Arithmetic







# How can you benefit the most from this course?

- Learn key abilities for future projects and your thesis
  - What are the type of questions I'll be able to answer?
  - What does it take to formulate a statistically sound answer to that question?
- Expand your critical thinking skills
  - ☐ How does knowledge come about?
  - ☐ How can I judge knowledge using my statistical reasoning skills?
- Understand some of the basics of Al
- ☐ Anything we should add?

**Make it relevant.** What is most relevant for you?

## Today

## **Topics**

Statistical reasoning with GLM

- Simple linear regression
  Simulation superpower
- Frequentist inference
  Course overview & organization

Multiple linear regression

Dummy-variable regression

Logistic regression

Multilevel and longitudinal analysis

Bayesian statistics

## **Learning goals**

Revisit simple linear regression, which is at the core of this course.

Get acquainted with statistical simulations and see how they can help elucidate statistical concepts.

Revisit some of the concepts of MvO1.

# Simple linear regression



## IJskoud de Beste

Joop

Can I use the weather forecast to predict the amount of ice cream to make?

What is the relationship between temperature and ice cream sales?

Me

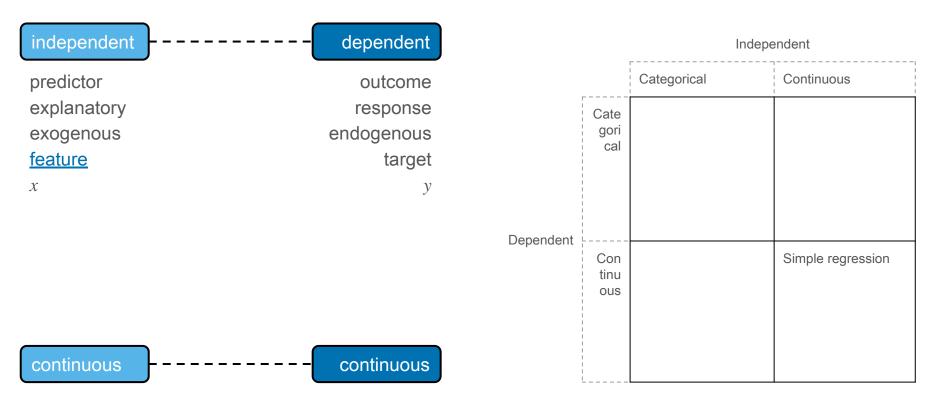


Photo by IJskoud de Beste

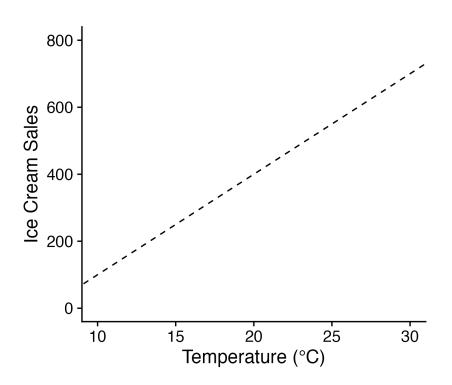


sales

# Conceptual model | Relationships between variables

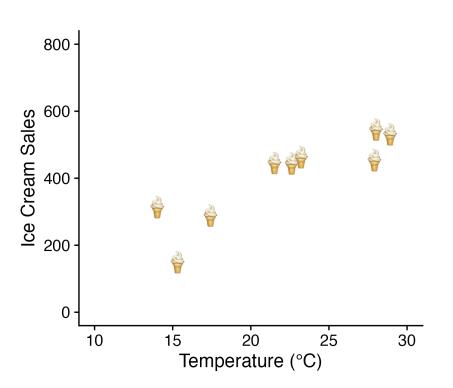


# Functional form | Linear relationships between variables



Sales = -200 + 30 Temperature

## Data collection

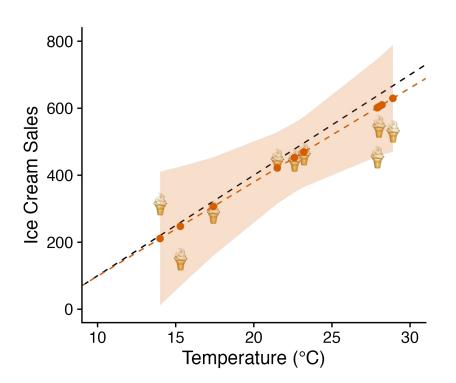


```
ice cream data <- tibble::tribble(</pre>
  ~temperature, ~ice cream sales,
  27.9,
                 452,
  15.3,
                 148,
 17.4,
                 287,
 21.5,
                 444,
 28.2,
                 935,
 14,
                 312,
  28,
                 544,
 28.9,
                 529,
  23.2,
                 461,
  22.6,
                 442)
```

ice\_cream\_sales =  $\alpha + \beta_1$ (temperature) +  $\epsilon$ 

```
mod <- ice_cream_sales ~ temperature # y ~ x
```

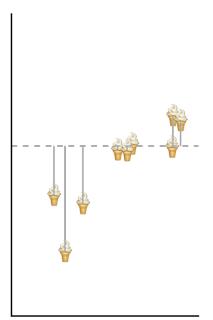
# Modeling | Linear regression

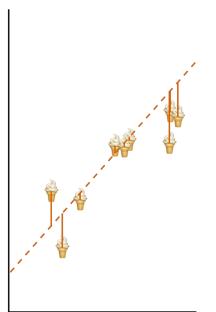


```
fit <- lm(mod, data = ice_cream data)
summary(fit); confint(fit, level = .95)</pre>
   Call:
   lm(formula = model, data = data)
   Residuals:
       Min
                10 Median
   -149.46 -89.73 -15.06
                             14.62 325.12
   Coefficients:
               Estimate Std. Error t value Pr(>|t|)
   (Intercept) -182.199
                                    -0.923
   temperature
                28.088
                                             0.0106 *
   Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
   Residual standard error: 142.7 on 8 degrees of freedom
  Multiple R-squared: 0.579, Adjusted R-squared: 0.5264
                   11 on 1 and 8 DF, p-value: 0.01059
   F-statistic:
 For every °C increase, sales increase on average with 9
 to 48 ice creams.
ice\_cream\_sales = -182.2 + 28.09(temperature)
```

# Modeling | Ordinary least squares

Residual SS ( $\beta_1 = 0$ ) Residual SS ( $\beta_1 = 28$ 





Objective: minimize residual sums of squares (RSS)

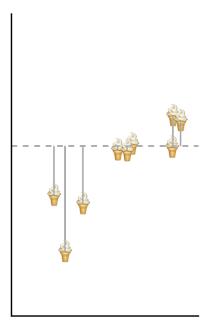
Baseline RSS  $Sales_i = \beta_o + e_i$ : 387052.4 Model RSS  $Sales_i = \beta_o + \beta_1$  Temperature<sub>i</sub> + e<sub>i</sub>: 162946

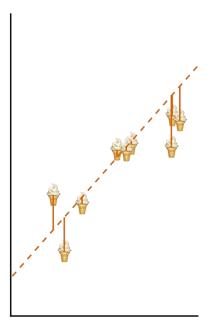
```
mod_baseline <- ice_cream_sales ~ 1
fit_baseline <- lm(mod_baseline, data = data)
summary(fit_baseline)

deviance(fit)
deviance(fit_baseline)</pre>
```

# Evaluation | How well does the model describe the data?

Residual SS ( $\beta_1 = 0$ ) Residual SS ( $\beta_1 = 28$ 





On average, how far off is my regression estimate from the observed values?

Standard deviation of the residuals, or root mean squared error (RMSE):  $\sqrt{(RSS / (n - K))}$ 

Model RMSE: 142.717 ice creams off

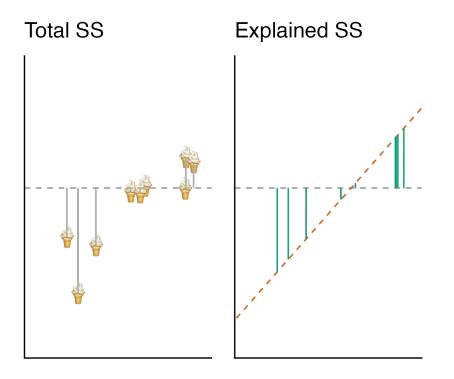
Is this good or bad?

Baseline RMSE: 207.379

Still, is it good or bad? Prediction!

summary(fit)\$sigma
summary(fit\_baseline)\$sigma

# Evaluation | How well does the model describe the data?



What is the proportion of variance shared by the data and the model?

Coefficient of determination (R<sup>2</sup>), or 'explained' variance: ESS / TSS

Model R<sup>2</sup>: 0.579

58% of variation in sales is shared with the model (i.e., temperature)

 $R^2$  = 1 (model fits data perfectly)

 $R^2 = 0$  (model doesn't do better than mean)

summary\_fit\$r.squared

# Evaluation | Is the relationship statistically significant?

Global significance (full model): *F* test

$$H_0$$
:  $R^2 = 0$   
 $H_1$ :  $R^2 = 0.579$  (fitted  $R^2$ )

See next lectures.

Local significance (one coefficient): t value

$$H_0$$
:  $\beta_1 = 0$   
 $H_1$ :  $\beta_1 = 28.088$  (fitted  $\beta_1$ )

See simulation superpower.

© Compute *t*-statistic for  $\beta_1$  (same procedure as for the mean): t = (28.088 - 0) / 8.468 = 3.317

# Evaluation | What is the size of the effect?

## Global effect size (full model)

 $R^2$  (correlation family, variance explained)

Cohen's  $f^2$  (correlation family, variance explained)

- $f^2 \ge .02 \text{ small}$ ; .15 medium; .35 large; Cohen (1988)

## Local effect size (one coefficient)

Cohen's  $f^2$  (correlation family, variance explained)

# Evaluation | Sign, size, significance, smelt

For every °C increase, sales increase on average with 9 to 48 ice creams.

On average, the model estimate is 142.717 ice creams off from the observed values.

§ 58% of variation in sales can be attributed to the model; F(1, 8) = 11.003, p = .011 (alpha = .05).

The slope of temperature differed significantly from zero; t(8) = 3.50, p = .011 (two-tailed, alpha = .05).

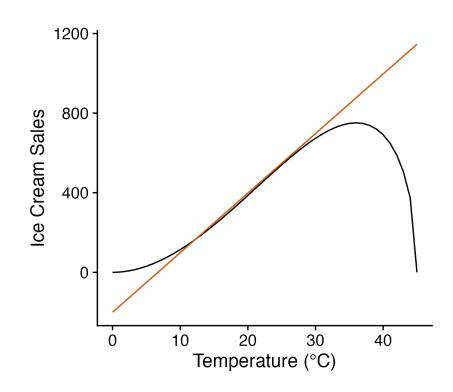
§ Effect size f is 1.375, which is considered large (Cohen, 1988).



## **Prediction**

- ☐ Linearity assumption
- Evaluate model on new data

```
new_data <- tibble(temperature = c(-10, 0, 10, 20, 30, 100))
predict(fit, newdata = new_data)</pre>
```



# Model thinking

What is the relationship between temperature and ice cream sales?

## **Conceptual model**

temperature

sales

## **Linear regression equation**

$$Sales_i = \beta_o + \beta_1 Temperature_i + e$$

#### R formula

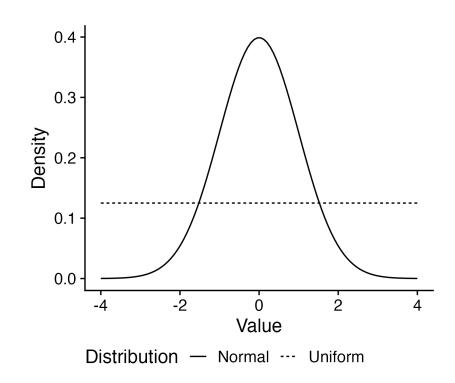
mod <- ice\_cream\_sales ~ temperature</pre>

# Simulation Superpower



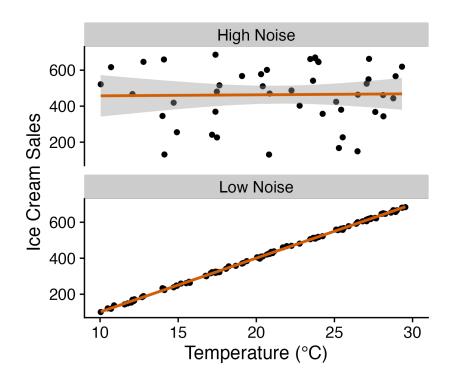
# Simulation #1 | Ice cream data

```
set.seed(0)
n < -10
temperature <- runif(n = n, min = 10, max
= 30)
intercept <- -200
slope <- 30
noise sd <- 120
ice cream sales <- intercept + slope *</pre>
temperature + rnorm(n = n, mean = 0, sd =
noise sd)
data <- tibble(temperature,</pre>
ice cream sales)
```



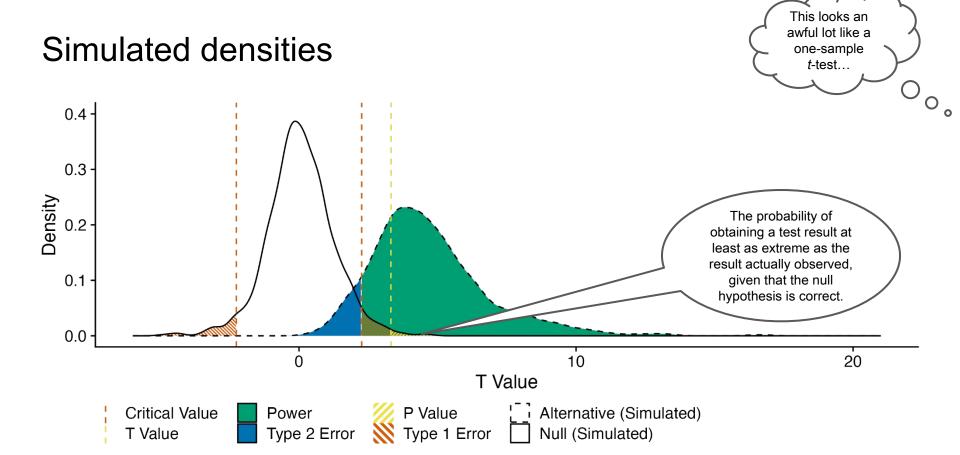
# Simulation #2 | High noise, low noise

```
noise sd low <- 5
ice cream sales sd low <- intercept +</pre>
slope * temperature + rnorm(n = n, mean =
0, sd = noise sd high)
noise sd high <- 500
ice cream sales sd high <- intercept +
slope * temperature + rnorm(n = n, mean =
0, sd = noise sd high)
```

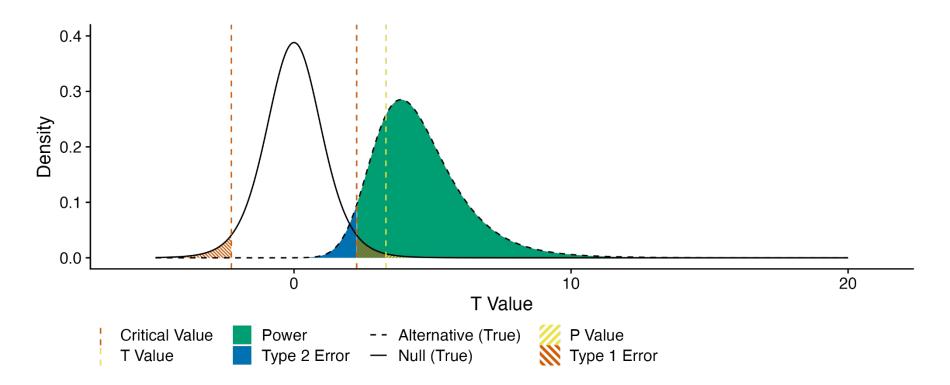


# Simulation #3 | Frequentist inference (NHST)

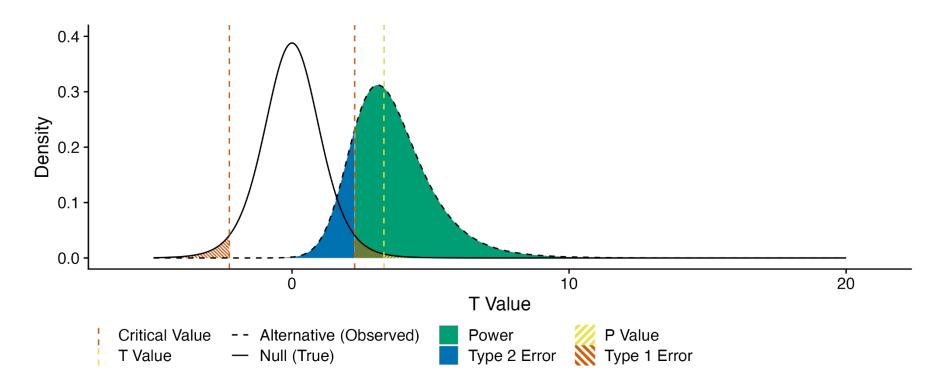
```
n sim <- 1000
slope null <- 0</pre>
slope alt <- slope
null t stats <- numeric(n sim)</pre>
for (i in 1:n sim) {
  obs temperature \leftarrow runif(n = n, min = 10, max = 30)
  ice cream sales <- intercept + slope null * obs temperature
  noise \leftarrow rnorm (n = n, mean = 0, sd = noise sd)
  data <- tibble(
    temperature = obs temperature,
    sales = ice cream sales + noise)
  fit <- lm(sales ~ temperature, data = data)
  null t stats[i] <- summary(fit)$coefficients[2, "t value"]</pre>
```



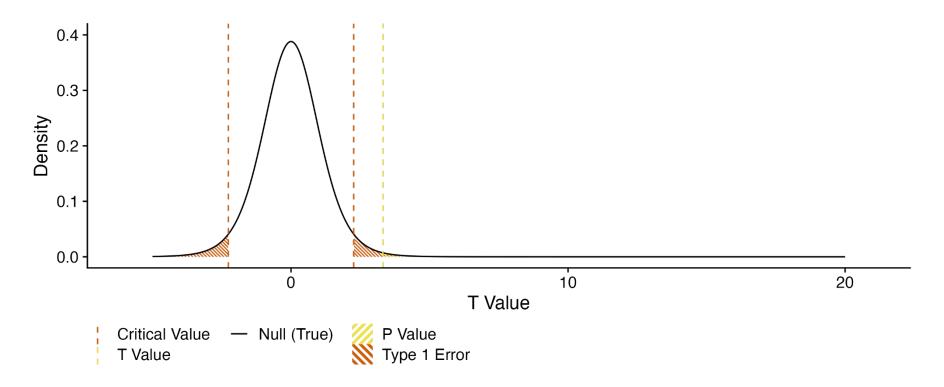
## True densities

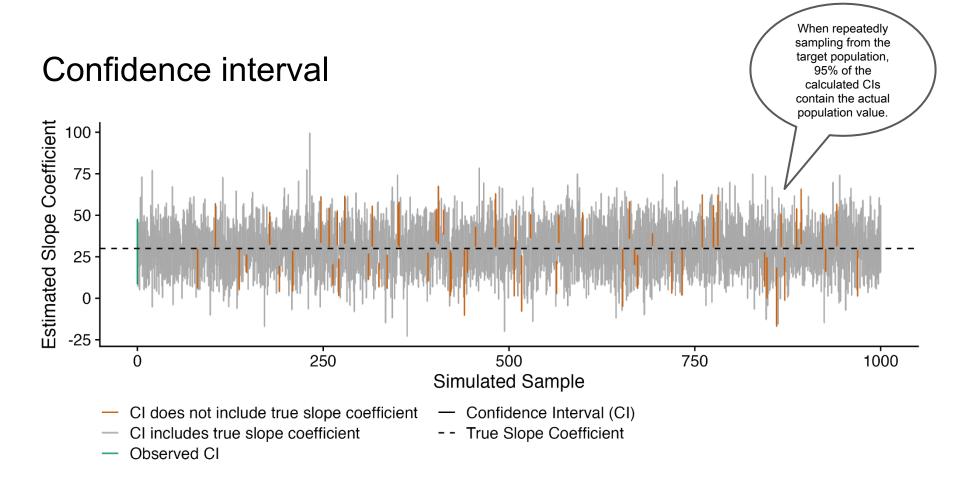


## Observed densities



# NHST in practice





# Introduction to R



## Best practices

- ☐ Create a <u>new project</u> for this course
- Use scripts and save regularly
  - □ .R
  - □ .Rmd (<u>R Markdown</u>)
  - □ .qmd (<u>Quarto</u>)
- ☐ Use <u>comments</u> excessively
- Clear history
- Understand pipes
  - function(function(x))
  - ☐ function(x) |> function()
  - ☐ function(x) %>% function()

**1.4.1** 38



## Book

## Applied Statistics Using R

- Builds on Chapter 18 of Whitlock & Schluter (where you left last year)
- Includes R examples for both frequentist and Bayesian analyses
- Not only for social sciences
- ☐ Check the <u>online resources</u>
- Don't pay more than 40 euro
- Rent or buy eBook: <u>VitalSource</u> (PayPal/creditcard only)
- Use it as a reference during the the remainder of your study

## Active reading

- Use <u>skimming</u> for some chapters, so you can scan them when there's something you really need to know.
- Use <u>intensive reading</u> (or 'study') for others (check online resources for executing R code).

Prepare the literature before the start of the week.

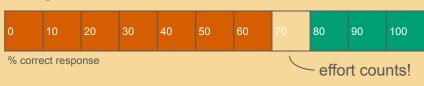


## What will this course add?

Week	Торіс	Literature	
		Skimming (R Programming)	Intensive reading (Statistical Inference)
1	Statistical reasoning with GLM	Revisit Chapter 18 from Whitlock & Schluter. Preface and Chapter 1, 2, 3.	Preface sections "How To Use This Book" and "Regression Approach to ANOVA". Chapter 7.
2	Multiple linear regression	Chapter 4, 5.	Chapter 8, 10.1, 10.2, 10.4, 10.7.
3	Dummy-variable regression	Chapter 6.	Chapter 9, 10.3, 10.5, 10.6.
4	Logistic regression		Chapter 11.
5	Multilevel and longitudinal analysis		Chapter 12.
6	Bayesian statistics		Chapter 15.
7	Surprise		

## Weekly assignments

#### Assignment sufficient, if at least 80% correct



#### Pass weekly assignment requisite, if at least 5 sufficient



# sufficient assignments

#### Catch-up insufficient assignments after exam



<sup>1</sup>if exam/retake grade is sufficient

## Online consultation (drop-in): every Wednesday 11:00–12:00

## I. Attempt problem independently

- Consult official resources (literature, lectures)
- This is what will be examined

#### II. Consult fellow students or teachers

- Collaborate and discuss with peers
- Attend lecture & ask questions
- Join the weekly drop-in consultation
- Active engagement will increase your success

## III. Use <u>UvA AI chat</u>\* responsibly

- Only if I & II are satisfied
- Treat as a coach, not as a solver
- Verify and reflect, beware of plagiarism
- Prediction ≠ intelligence

<sup>&</sup>lt;sup>2</sup>you must contact Alexander & Jonas to participate

<sup>\*</sup> Available this academic year

## Module organization\*

(\*course manual is leading)

#### Team

- ☐ Alexander Savi (lecturer, coordinator)
- Jonas van Nijnatten (assignments, co-coordinator)
- Sytske Schep and Tim Vervenne (teaching assistants)

#### Contact

- General: during lectures and through psychobiologiejaar2-science@uva.nl.
- □ Statistical reasoning: during lectures.
- Weekly assignments: during walk-in consultation hours.

## **Assignments (formative)**

- Weekly, sufficient/insufficient, deadline every Sunday 23:55 (correct answers available after deadline)
- □ Pass with 5/7 sufficient assignments
- □ 3 attempts per assignment, 2 prior checks per attempt
- Exemption for recidivists (if previously sufficient) brand new topics!
- ☐ Use decimal *points* (.)

## **Exam** (summative)

- 80% of final grade (5% SR, 3% PhS)
- SR digital open book (PhS *not* open book)

# Cooling Down



## Takeaways



Illustration by **Zach Weinersmith** 

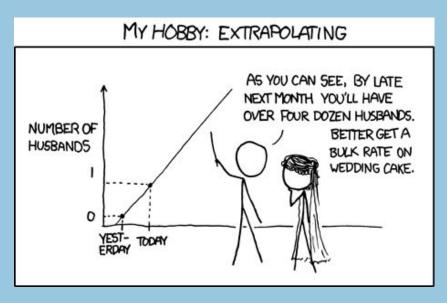
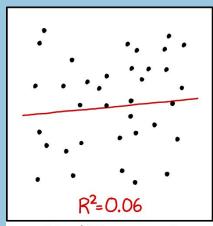
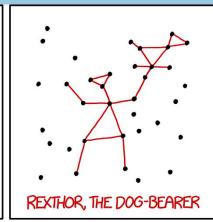


Illustration by Randall Munroe (wtf)

## **Takeaways**





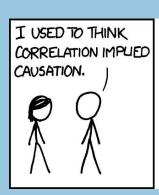
I DON'T TRUST LINEAR REGRESSIONS WHEN IT'S HARDER TO GUESS THE DIRECTION OF THE CORRELATION FROM THE SCATTER PLOT THAN TO FIND NEW CONSTELLATIONS ON IT.

Illustration by Randall Munroe (wtf)

## Takeaways



Illustration by Timo Elliott



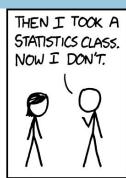




Illustration by Randall Munroe (wtf)

47



## Don't look here!

Show the distribution of *p*-values when the null-hypothesis is true (e.g., two samples come from the same population).

Additional challenge I: what happens to the distribution if the null hypothesis is false?

Additional challenge II: argue why the distribution in the comic would be problematic.

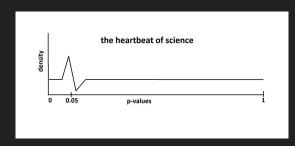


Illustration by František Bartoš

Hints (select and copy/paste the invisible text below to reveal it)

0.

1.

2.

3.



## Slides

alexandersavi.nl/teaching/

## License

Statistical Reasoning by Alexander Savi is licensed under a <u>Creative Commons</u>
<a href="https://doi.org/10.1007/j.nc/4.001/j.nc/4.00