Complex systems: week 3

Mathematical Psychology

Han van der Maas

Assignment

- option 1:
- Make your own cusp model for a psychological transition
  - choose behavioral and control variables
  - check flags to see whether model makes sense

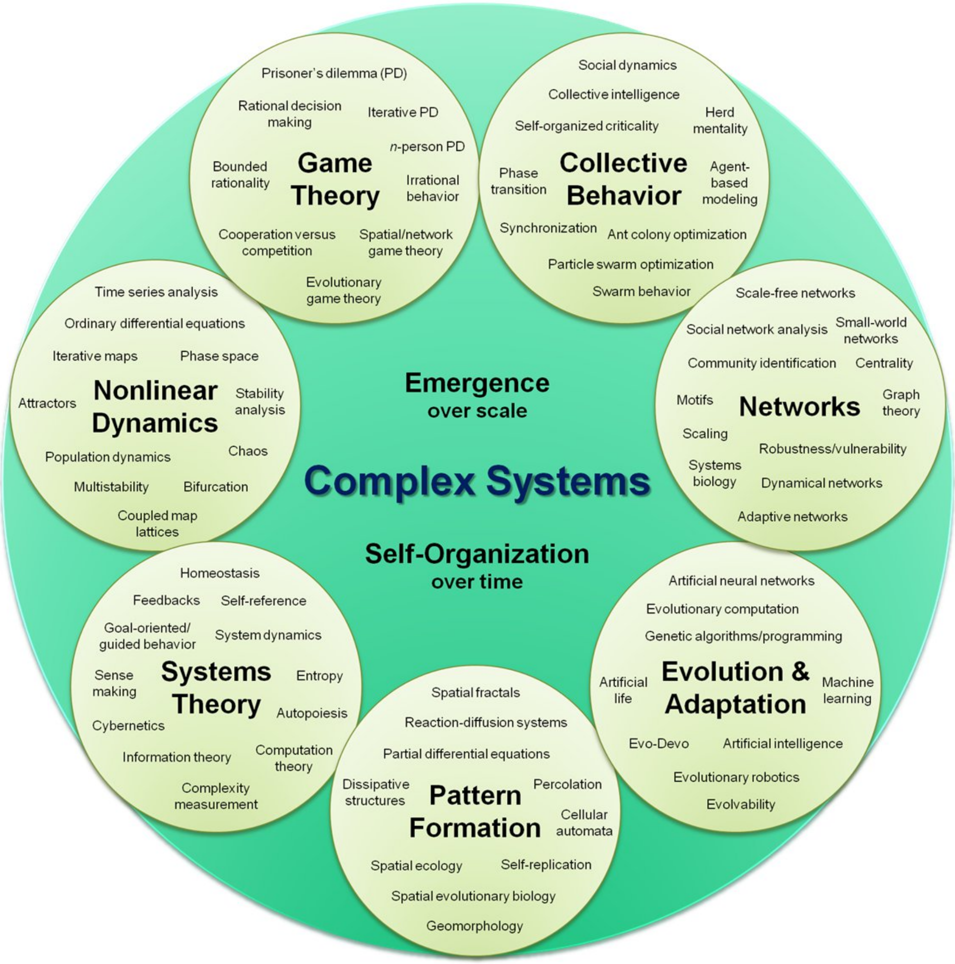
- option:
- Build Zeeman Machine
- collect data
- fit in R with cusp.fit()

The human mind is (the ultimate) complex system

- We are studying *developing* complex systems
- I use formal models & methods from complex system research in natural sciences

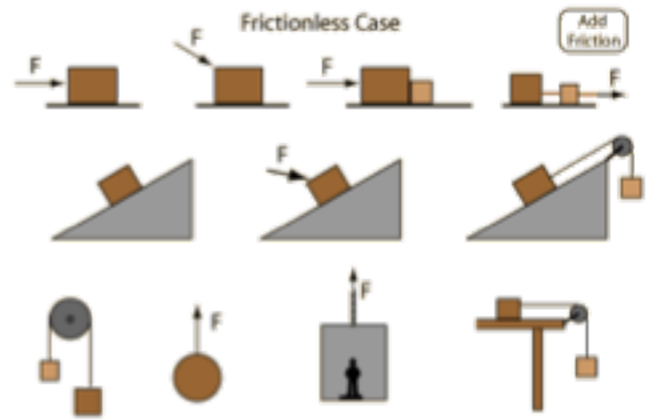
- Developmental examples

1. Phase transitions
2. Networks: the wiring of cognition
3. Novel measurement: Child practice-monitoring systems



Naïve psychologist view of Physics

- linear?
- reductionism?
- simple systems?
- Newtonian mechanics?

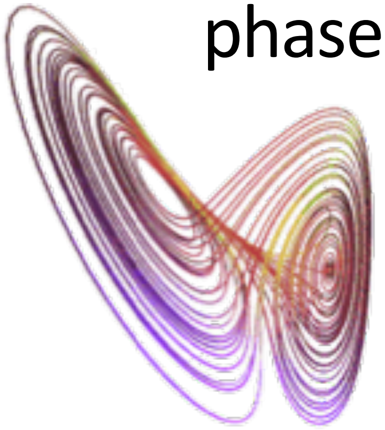


- Complexity, nonlinearity, self-organization

Natural science

- many systems are open complex systems consisting of many sub-systems interaction nonlinearly
- many models, tools and techniques have been developed to study such systems
- and applied in physics, chemistry, biology etc.

- Behavior
  - Pattern formation
  - Self-organizing
  - Strange attractors
  - Unpredictable (chaotic)
  - Instabilities
  - Bifurcations and phase transitions



- Methods
  - Mathematical modeling (differential and difference equations)
  - Simulation
  - Perturbation analysis
  - Analytical and numerical bifurcation analysis

Social science

- No equations
  – A few nice exceptions: Murray & Gottman model of marriage, Helbing model of panic, neural models of reaction time
- But humans are complex, instable, nonlinear etc.
- How to proceed?
  – metaphorical
  – qualitative analysis
  – statistical
  – making mathematical models

But first an introduction

- Main phenomena
  - Deterministic Chaos
  - Self-organization
  - Catastrophes

Part 1: Chaos

- Seemingly random behavior in deterministic nonlinear systems
- small changes in initial conditions lead to large changes in the future (Butterfly effect)
  - http://www.youtube.com/watch?v=Qe5Enm96MFQ

dynamical systems

- difference equation (map)
  - $x(t+1)=f(x(t),p)$
- differential equation (flow)
  - $dx/dt=f(x,p)$
- if f is nonlinear, then NLDST
- if $x(t=0)$ is known, you like to compute $x(t)$ but then you have to solve the flow first
- This is often impossible when f is nonlinear

Verhulst

- Population growth
- rabbits on island
- model 1: x_{t+1}=a x_t
  - a=2; $x_{t=0}$=2
  - what happens
  - solution $x_t=x_{t=0}a^t$
- model 2: x_{t+1}=a x_t (K-x_t)/K
  - what happens when x_t<<K?
  - what happens when x_t is close to K?
  - what is the role of a?

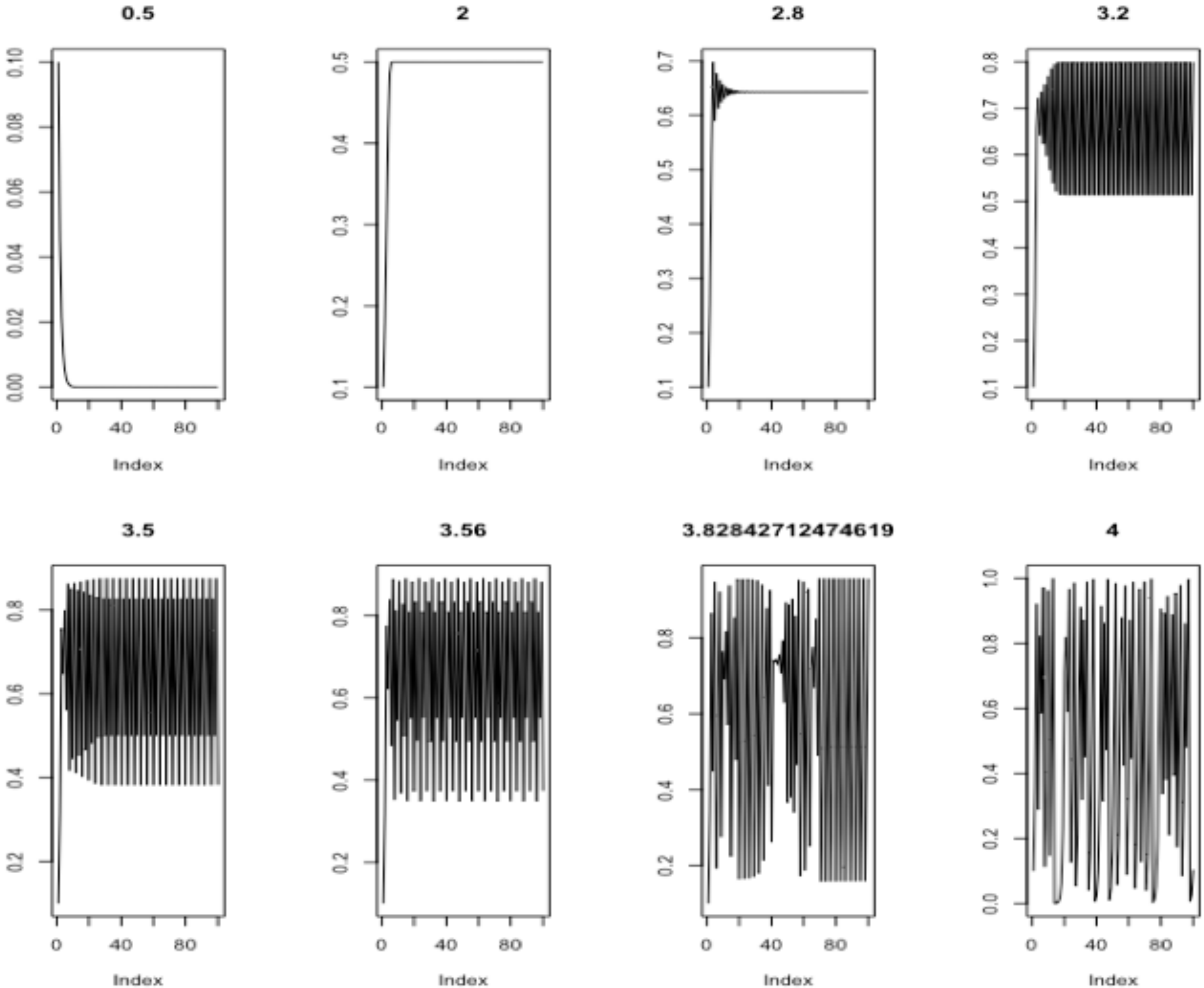
numerical analysis

- f=function(x,a) a*x*(1-x)
- a=2;x=.001
- for(i in 2:50) x[i]=f(x[i-1],a)
- plot(x,type='l',xlab='time',bty='n')

- fixed points?
- stability?

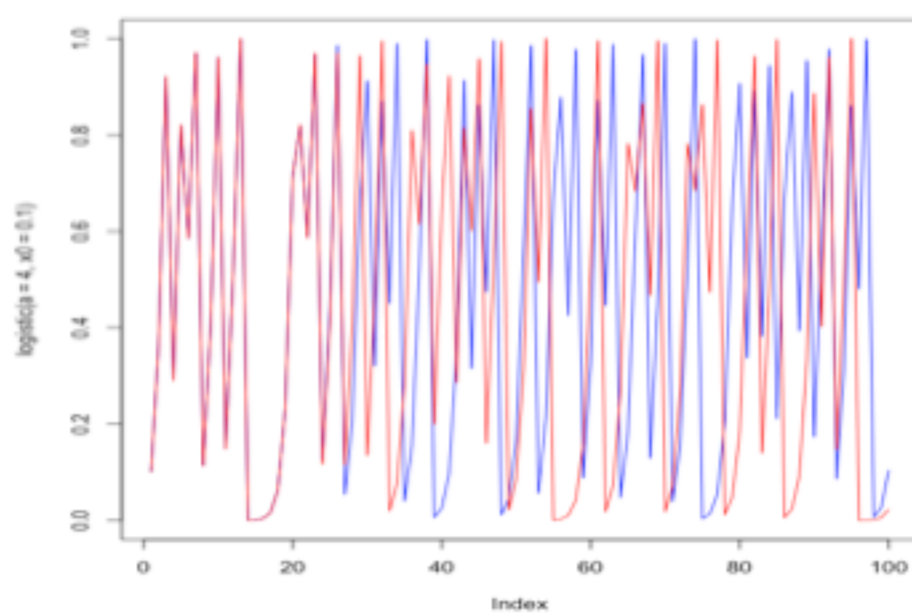
period doubling

- logistic= function(a=2,n=100,x0=.1)
- {f=function(x,a) a*x*(1-x)
- x=x0
- for(i in 2:n) x[i]=f(x[i-1],a)
- x}
- layout(matrix(1:8,2,4,byrow=T))
- a=c(.5,2,2.8,3.2,3.5,3.56,1+sqrt(8),4)
- for(i in 1:8) plot(logistic(a=a[i]),type='l',main=a[i],ylab='')


0.5
2
2.8
3.2
3.5
3.56
3.82842712474619
4
Index

chaos: initial conditions

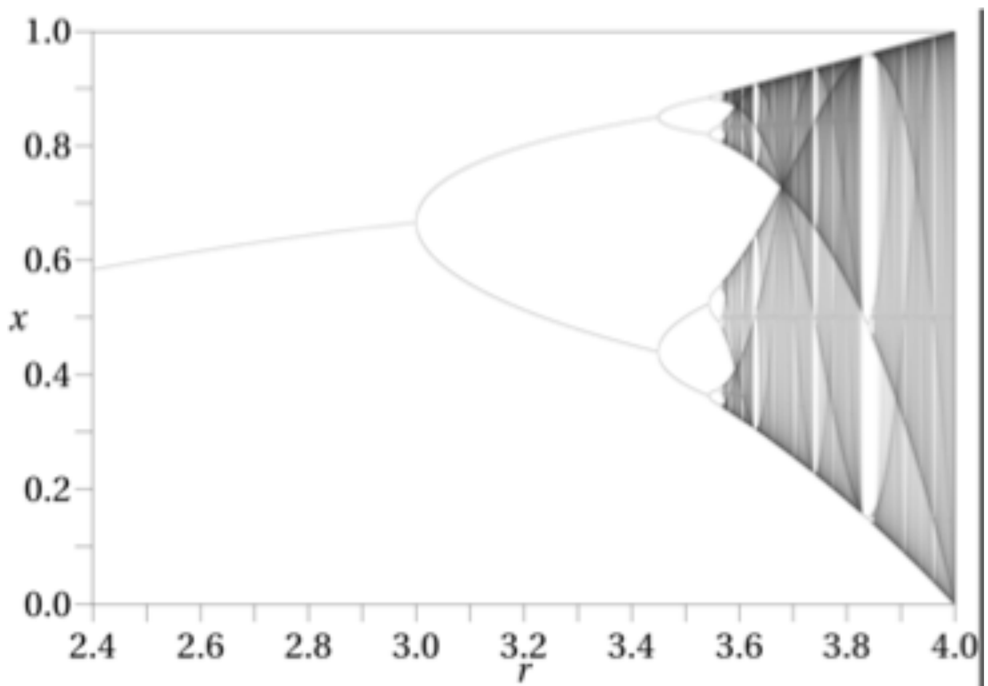
- Sensitivity to initial condition: Butterfly effect
- plot(logistic(a=4,x0=.1),type='l',col='blue')
- lines(logistic(a=4,x0=.100000001),col='red')



bifurcation diagram

- Map of equilibria as function of control parameter(s)

- a=seq(1,4,by=.1);
- k=length(a);
- m=matrix(,k,100)
- for(i in 1:k) m[i,]=logistic(a=a[i],n=500)[401:500]
- matplot(m,pch='.',col='black')



In psychology?

- Metaphorical application
- Sensitivity to initial conditions
- Chaos in the brain
  - More healthy more chaotic
  - Phase plot of time series (EEG)
  - Dimensionality (Hausdorff dimension)
    - The problem of noise
  - 1/f noise (long term correlations)
- ‘Philosophical’ relevance: deterministic world already unpredictable

Part 2: self-organization

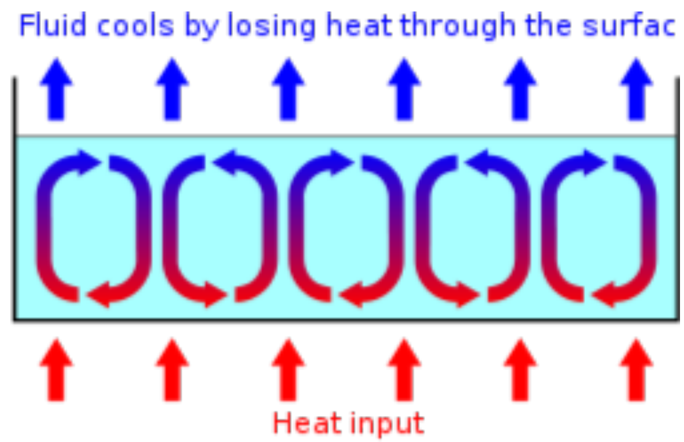
Self-organization

- Something is self-organizing if, left to itself, it tends to become more organized.
- Simple components with nonlinear local interactions leading to spatial or temporal organized behavior
- Open, feedback loops
- Nonlinear dynamics
- No central command
- http://www.scholarpedia.org/article/Self-organization

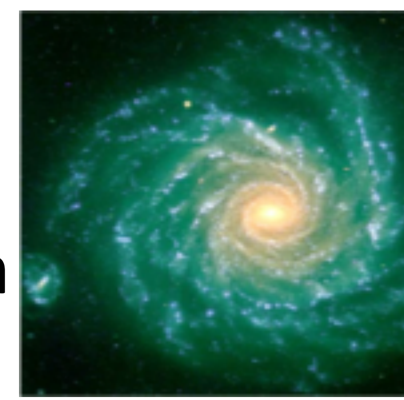
Laser, weather, spiral waves, chemical patterns, swarms, ants nests, brains, traffic jams

Physics

- Laser
- Turbulence

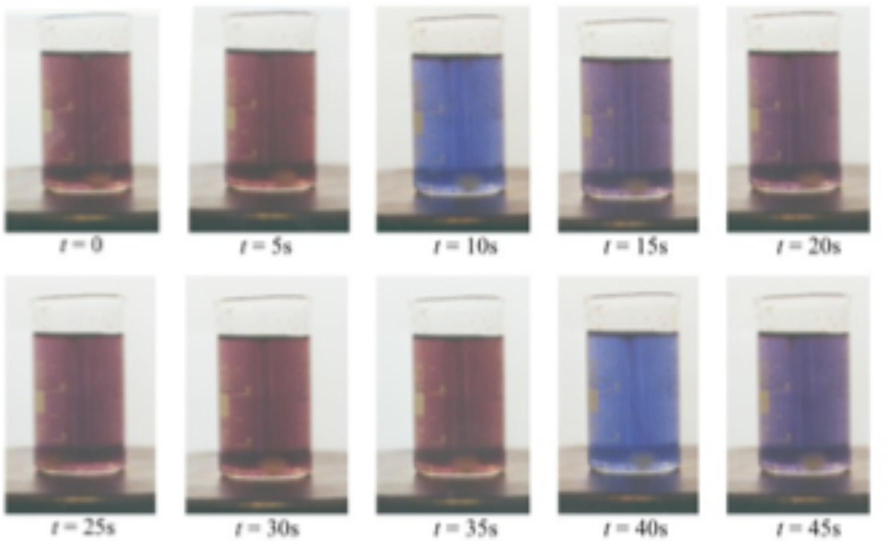


- http://www.uni-magdeburg.de/abp/picturegallery.htm



Chemistry: Belousov–Zhabotinsky reaction

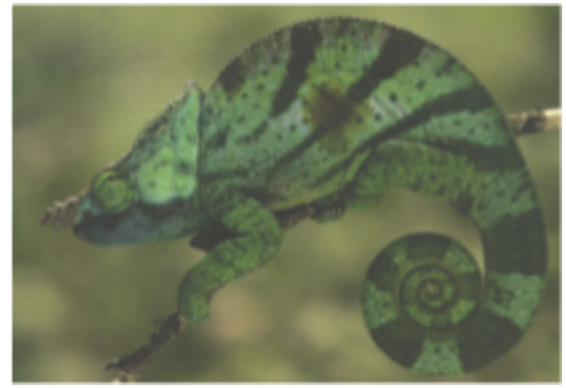
- Family of Oscillating chemical reactions (bromide and some acid)
  - Color oscillations
  - Pattern formation



- http://www.youtube.com/watch?v=IBa4kgXI4Cg

Biology

- Spiral waves
- Ecosystems
- Ant colonies
- Flocking



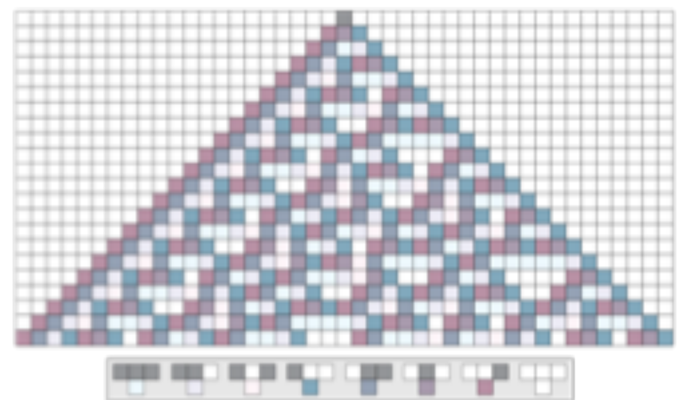
Famous players

- Physics: Herman Haken (synergetics)
- Chemistry: Belgium Nobel price winner Prigogine (non-equilibrium thermodynamics)
- Biology (Evolution): Kauffman (origin of order)
- Mathematics: Mandelbrot (fractals)
- Poincaré, Edward Lorenz, Feigenbaum, Floris Takens
- Famous labs: Sante Fe…

Simulation of SO: Cellular Automata

- Matrix or row of cells
- behavior cell depends on neighbors
- local rules
- global spatial effects
- Example: Wolfram code
- Netlogo: CA 1d Elementary
- Wolfram classes
  - Type 1: homegeneous (4)
  - Type 2: regular (24)
  - Type 3: chaotic (18)
  - Type 4: complex (110,54)

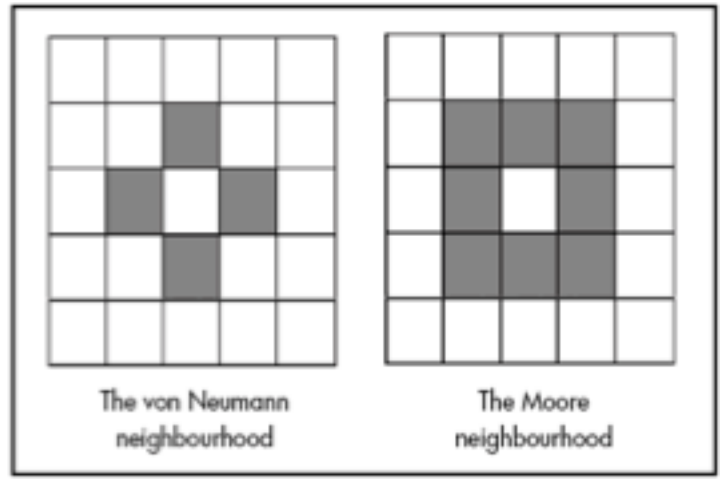
current pattern	111	110	101	100	011	010	001	000
new state for center cell	0	0	0	1	1	1	1	0



Conway's Game of life: 2D CA

- Rules:
  - Less than 2 neighbors: die
  - More than 3 neighbors: die
  - Empty and three neighbors: live
- Choose a seed and watch!

- http://www.ibiblio.org/lifepatterns/



Extensions

- Artificial life
- Multi agent systems
- Neural networks
- Robotics
- Genetic algorithms

netlogo

- NetLogo is a cross-platform multi-agent programmable modeling environment.
- download netlogo or run in your browser

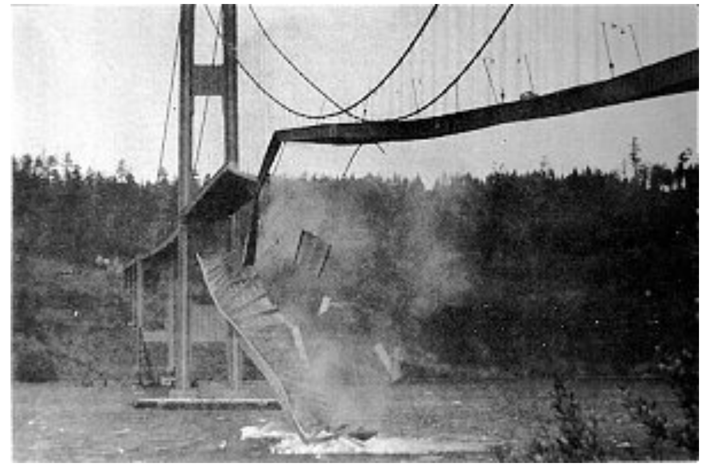
- examples: ca, flocking, sunflower
- don’t try the sound machine !

Self organization in psychology

- Neural networks
  - Synchronization
- Social patterns
  - Ethnocentricity in Netlogo
  - Panic (escape)
  - Traffic jams
- Clinical disorder
  - Attractors in networks of symptoms
- Gestalt like perception
- Movement science

Part 3: Catastrophe theory

- Branch of dynamical system theory
- Qualitative (topological) models of bifurcations
- Detect and model phase transitions in order to characterize the system qualitatively
- Subsequently, the qualitative different states can each be modeled with standard (statistical) quantitative methods
- Self organization demarcated by phase transitions



Phase transition

- Is not an acceleration
- Is not a sudden change caused by a large change in independent (control) variables

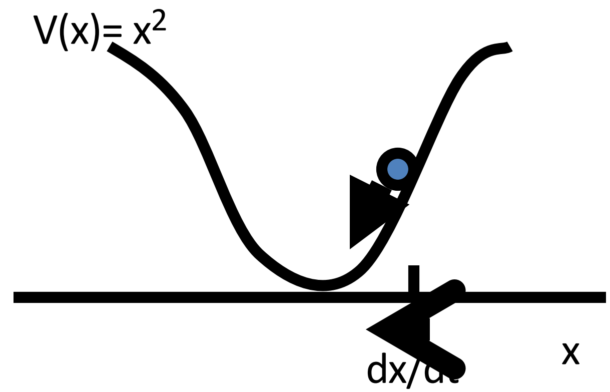
- A smooth continuous change in the control variable leads to a sudden jump

Attractor landscape

- Phase transition can occur when the attractor landscape changes (i.e. number and/or type of fixed points change)
- The current attractor (stable fixed point) should become unstable
- At this point the fixed point is ‘degenerate’

Degenerate critical points

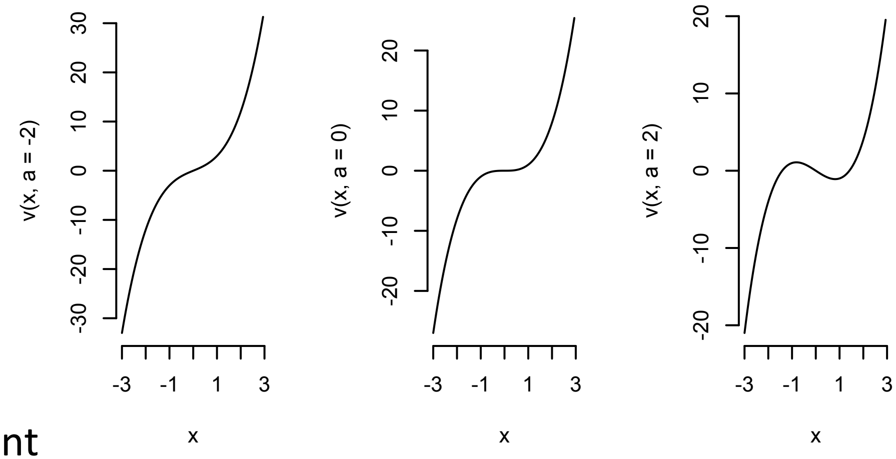
$$\frac{dx}{dt} = -V'(x;c)$$



- Change in x over time is defined by the shape of potential function
- Critical point if first derivative is zero
- Degenerate if first and second derivative are zero (compare x^2 with x^3)

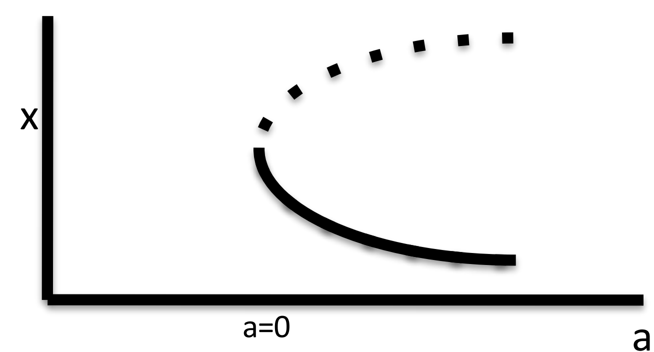
$V=x^2$ versus $V=x^3$

- $V=x^2-a*x$, $V'=2x-a$, $V''=2$ (never zero)
- $V=x^3-a*x$, $V'=3x^2-a$, $V''=6x$; $V'(0)=0$ & $V''(0)=0$
- $V=x^3-a*x$ is fold catastrophe



- layout(t(1:3))
- v=function(x,a) x^3+a*x
- curve(v(x,a=2),-3,3,bty='n')
- curve(v(x,a=0),-3,3,bty='n')
- curve(v(x,a=-2),-3,3,bty='n')

- So the fold has a degenerated critical point in (0,0) meaning that configuration (number and/or type) of equilibria changes.
- All kinds of degenerate critical points possible in dynamical systems

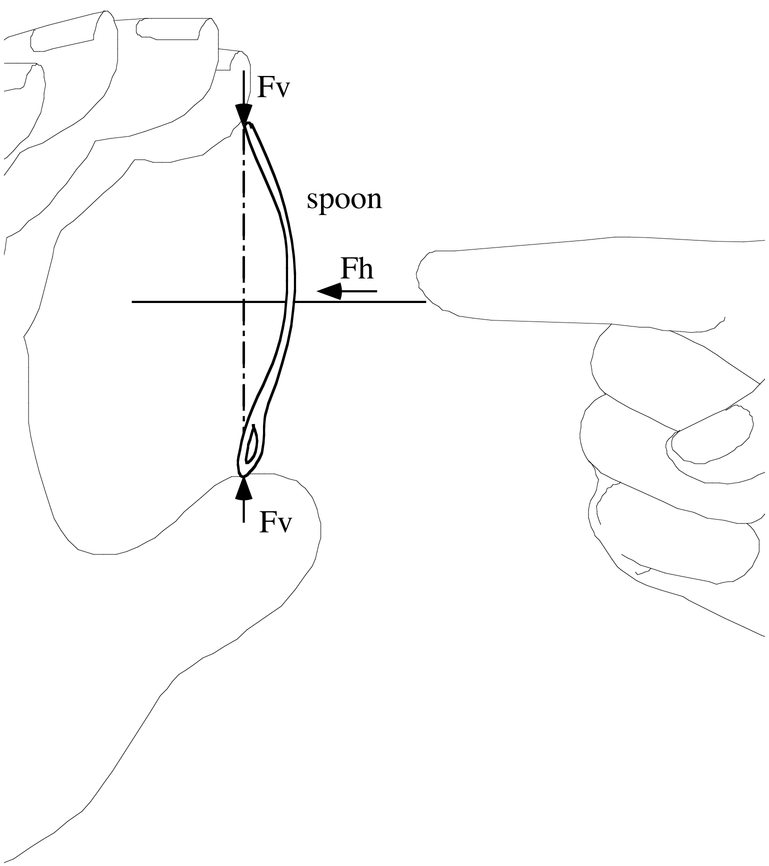


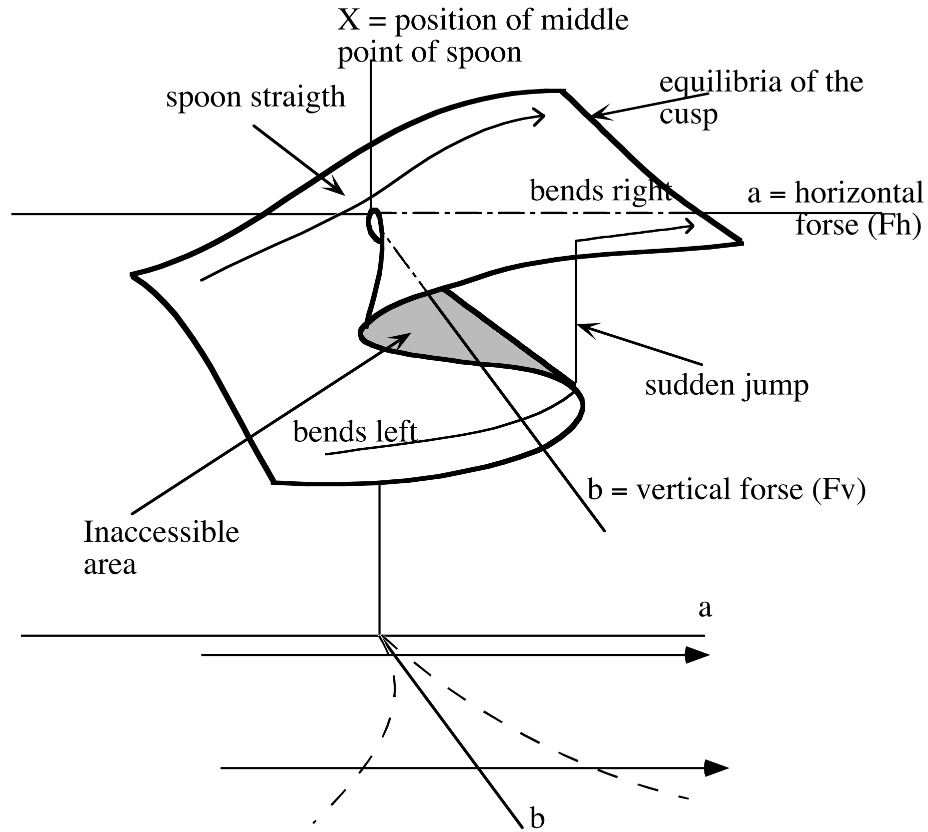
Elementary catastrophes

- René Thom proved that all structural stable systems (such as potential functions), given a limited set of control variables and given a set of very flexible scale transformations (diffeomorphisms), can be classified into 7 elementary catastrophes
- The simplest catastrophe showing discontinuous transitions is the cusp catastrophe

- Note: Recent applications in physics, chemistry, economics
- Note: Famous criticism of CT can be answered with statistical methods
- Note: CT incorporated in Bifurcation theory
- Note: CT handles point attractors, not limit cycles and strange attractors

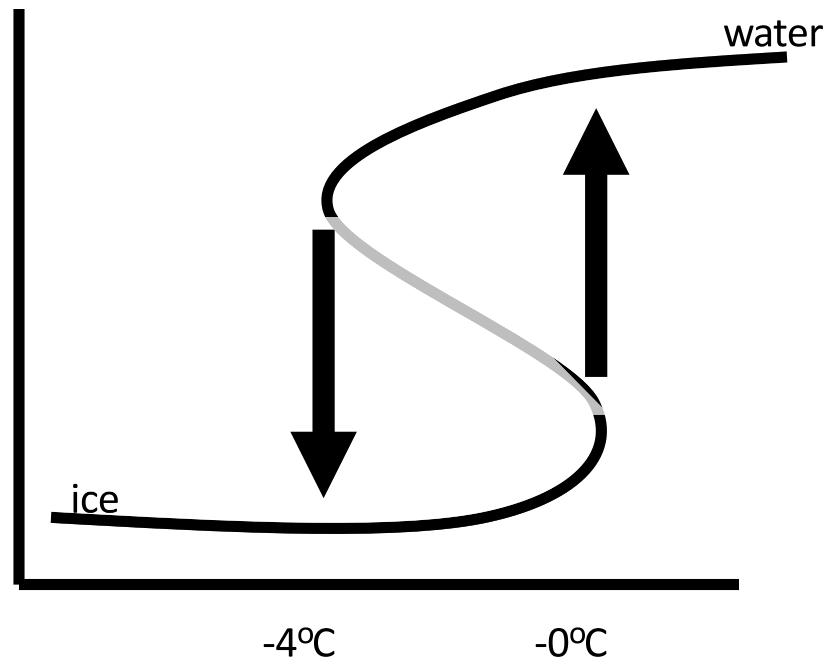
My favorite example





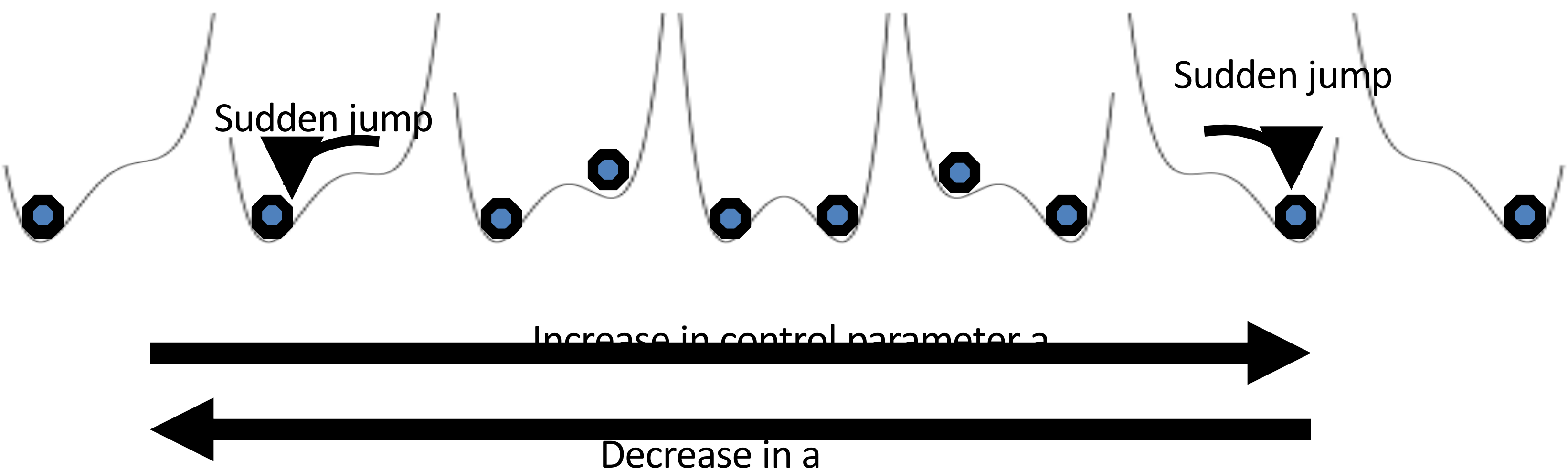
hysteresis

- ‘To lag behind’
- Property of phase transitions
- Example: in disturbance free conditions, water freezes at -4^{o}, and ice melts at 0^{o}c.
- What is the splitting axis?



Hysteresis again

A sudden jump between two different stable states by means of arbitrarily small continuous changes in the independent variables.



$$V = -\tfrac{1}{4}x^4 + \tfrac{1}{2}bx^2 + ax$$

Delay convention

- Noise in the system decreases hysteresis effects
- No noise (maximal hysteresis): Delay convention
- Lot of noise (no hysteresis) : Maxwell convention

Methods of applications

1. Mathematical
   - Idea is: take a system such as $dP/dt=KP(1-N/P)*(M-P)=MPK-MKP^2/N-MKP^2-KP^3/N$
   - See that highest power is P^3 so it is a cusp!
   - Reparameterize to a and b of cusp. Such that a=f(M,K,N), b=g(M,K,N) and you completely understand the system
2. Cusp fit
3. Flags

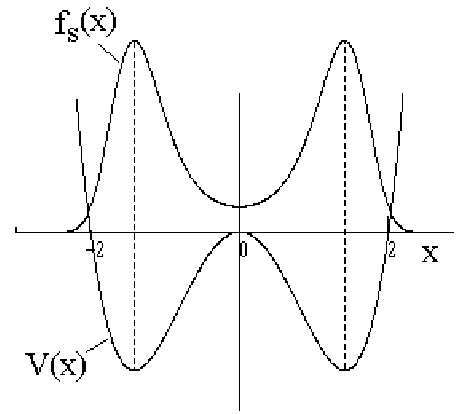
Cobb's ML technique

$$f(X \mid \alpha,\beta) = D \exp(-1/4\, X^4 + 1/2\beta X^2 + \alpha X)$$

$$X = (Y-\lambda)/\sigma$$

$$\alpha = a_0 + a_1 Z_1 + a_2 Z_2 + ..a_n Z_n$$

$$\beta = b_0 + b_1 Z_1 + b_2 Z_2 + ..b_n Z_n$$



- f is distribution function based on potential function V
- Z and Y are observed, D is integration constant
- The parameters λ, σ, $a_0..a_n$, and $b_0..b_n$ are estimated using a maximum likelihood

Grasman, R. P. P. P., van der Maas, H. L. J. & Wagenmakers, E.-J. (2009) Fitting the cusp catastrophe in R: A cusp-package primer. *Journal of Statistical Software, accepted*

Cusp package

- **install.packages("cusp", repos="[http://R-Forge.R-project.org")](http://R-Forge.R-project.org)**
- library(cusp)
- set.seed(1)
- x1 = runif(150)
- x2 = runif(150)
- z = Vectorize(rcusp)(1, 4*x1-2, 4*x2-1)
- data <- data.frame(x1, x2, z)
- fit <- cusp(y ~ z, alpha ~ x1+x2, beta ~ x1+x2, data)
- print(fit)
- summary(fit)

Parameters en fit

- Coefficients:

	Estimate	Std. Error	z value	Pr(>|z|)	
a[(Intercept)]	-2.49423	0.23223	-10.740	< 2e-16	***
a[x1]	3.99946	0.01819	219.911	< 2e-16	***
a[x2]	0.56217	0.33825	1.662	0.0965	.
b[(Intercept)]	0.20129	0.48895	0.412	0.6806	
b[x1]	-0.82708	0.80677	-1.025	0.3053	
b[x2]	3.39448	0.51275	6.620	3.59e-11	***
w[(Intercept)]	-0.10376	0.07287	-1.424	0.1545	
w[z]	1.10477	0.02648	41.724	< 2e-16	***

	R.Squared	logLik	npar	AIC	AICc	BIC
Linear model	0.5094301	-186.4804	4	380.9609	381.2367	393.0034
Cusp model	0.6839674	-111.5130	8	239.0261	240.0473	263.1112

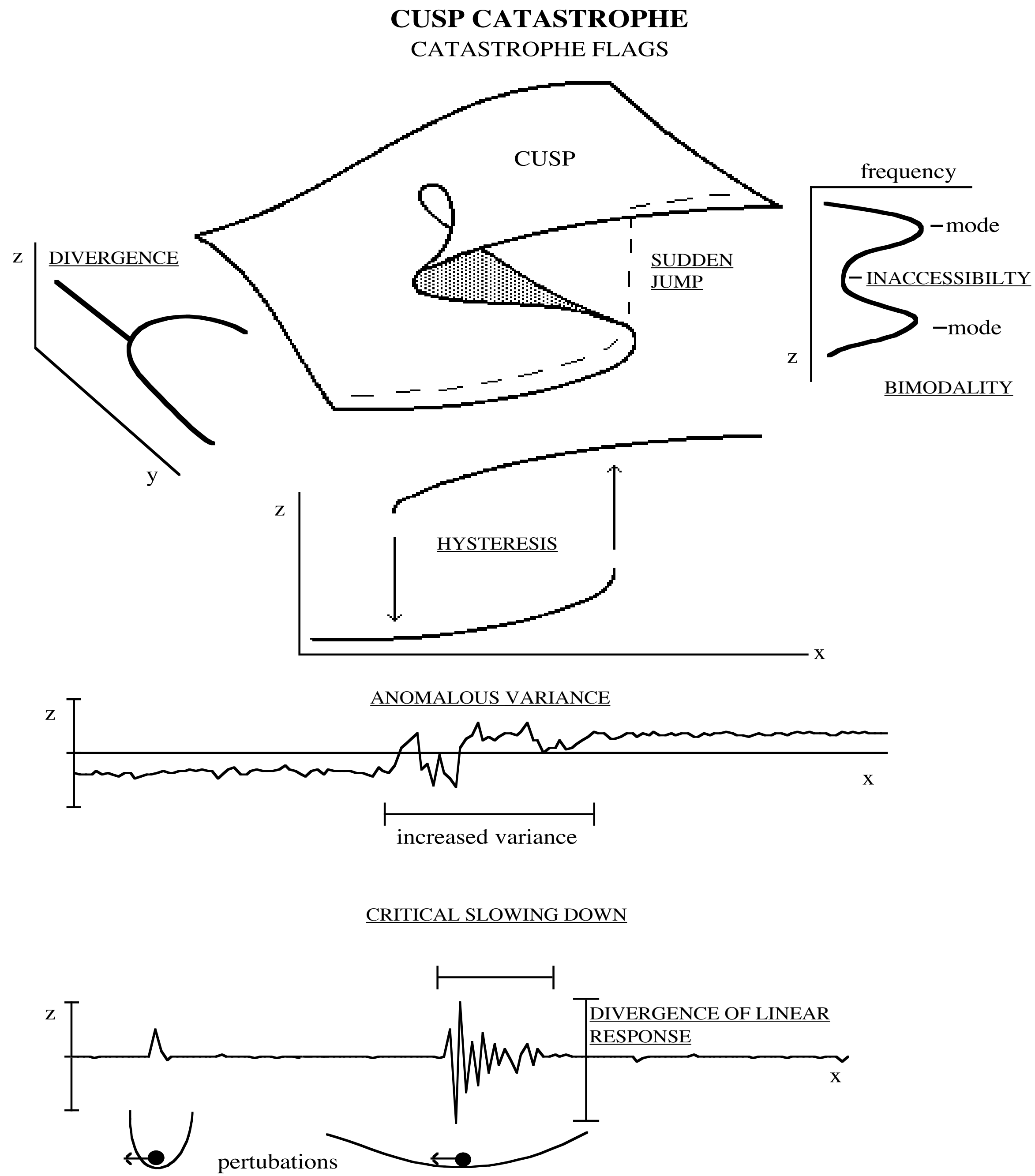
Zuiniger model

- Fout model:
  - fit2 <- cusp(y ~ z, alpha ~ x1, beta ~ x1, data)
  - summary(fit2)
  - BIC = 297.4 slechter
- Zuiniger model
  - fit3 <- cusp(y ~ z, alpha ~ x1, beta ~ x2, data)
  - summary(fit3)
  - BIC = 256.7 beter!

Cata-strophe flags

By detecting catastrophe flags in real data, more and more support for the discontinuity hypotheses can be established

Note: some are necessary (bimodality) others are perhaps sufficient (hysteresis)

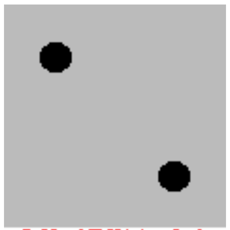


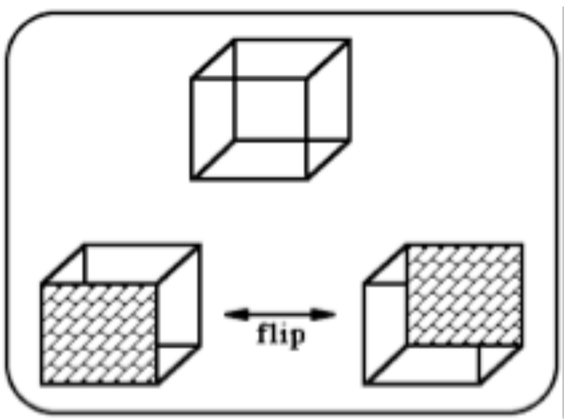
Psychological applications

- Detect flags
  - Start with multimodality
- Formulate cusp model
  - Choose control variables
- Fit cusp model
- Develop full mathematical model

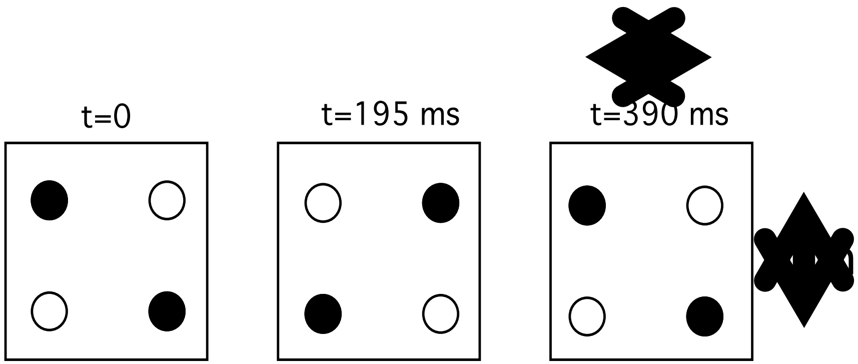
- Insight
- Anger
- Falling in love
- Multistable perception
- Religious experience
- War
- Relapse
- Suicide
- Quitting smoking
- Understanding the cusp catastrophe
- Learning
- Depression
- …

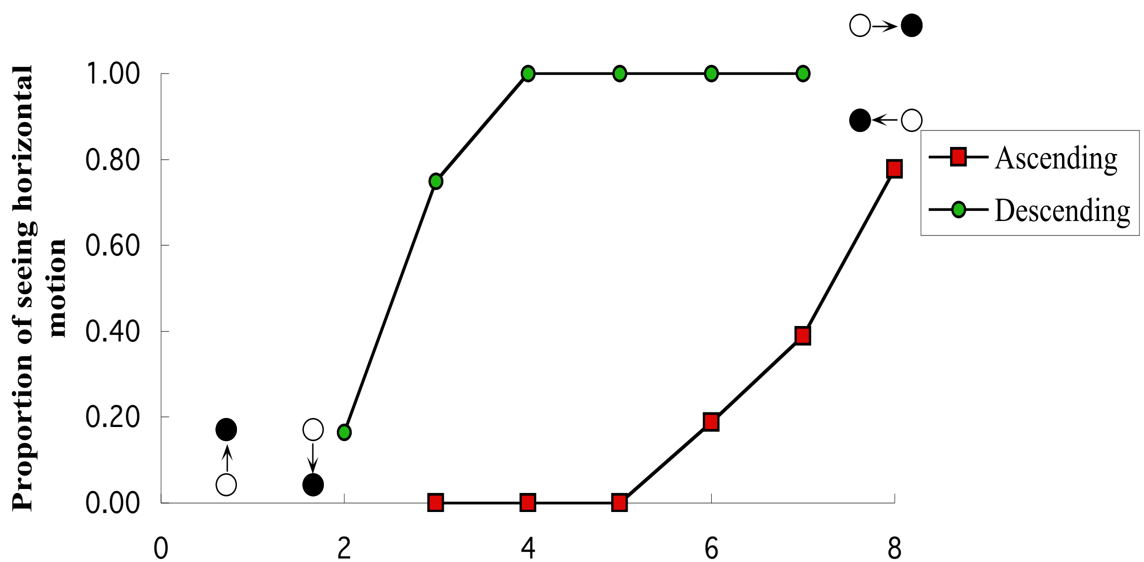
Multistable Perception





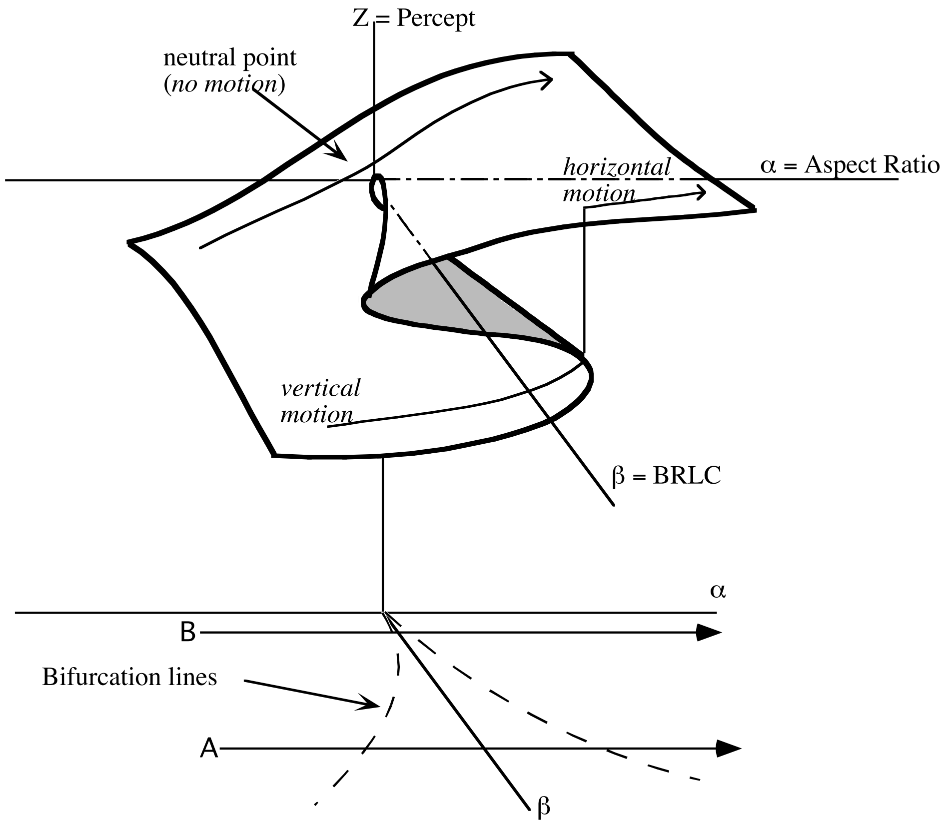
Apparent motion

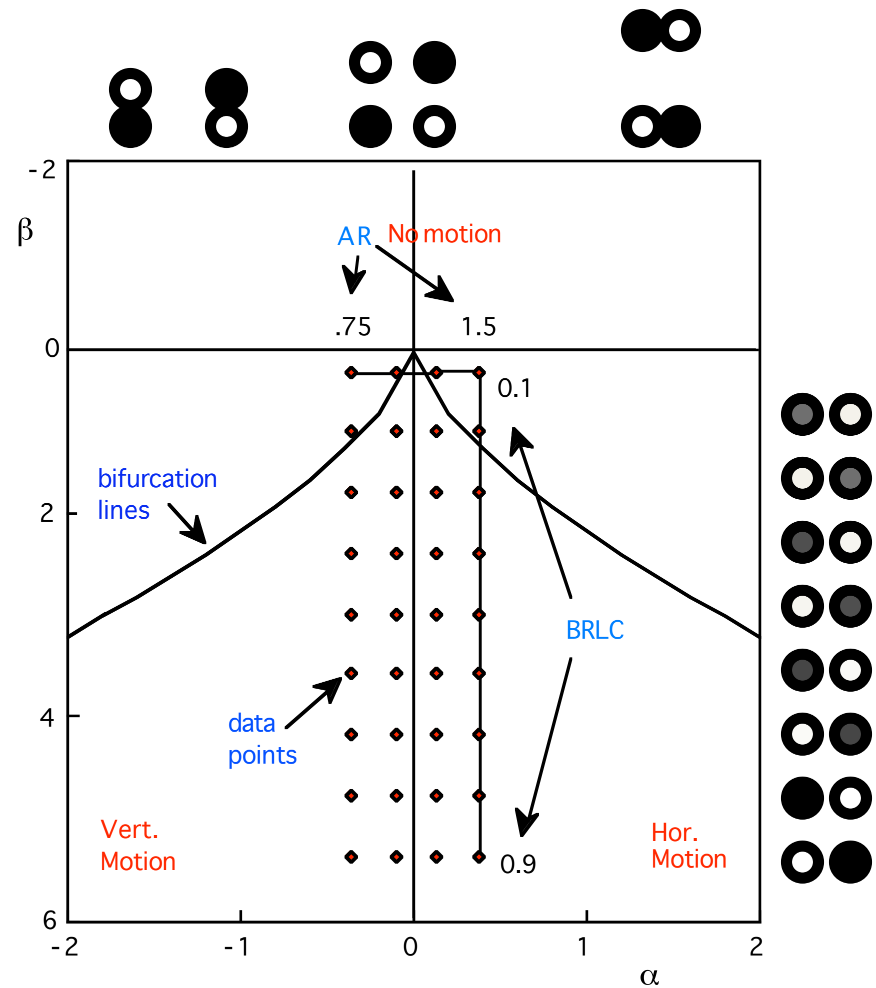




= h/w

Model & Fit


Z = Percept
neutral point
(no motion)
horizontal
motion
α = Aspect Ratio
vertical
motion
β = BRLC
α
B
Bifurcation lines
A
β

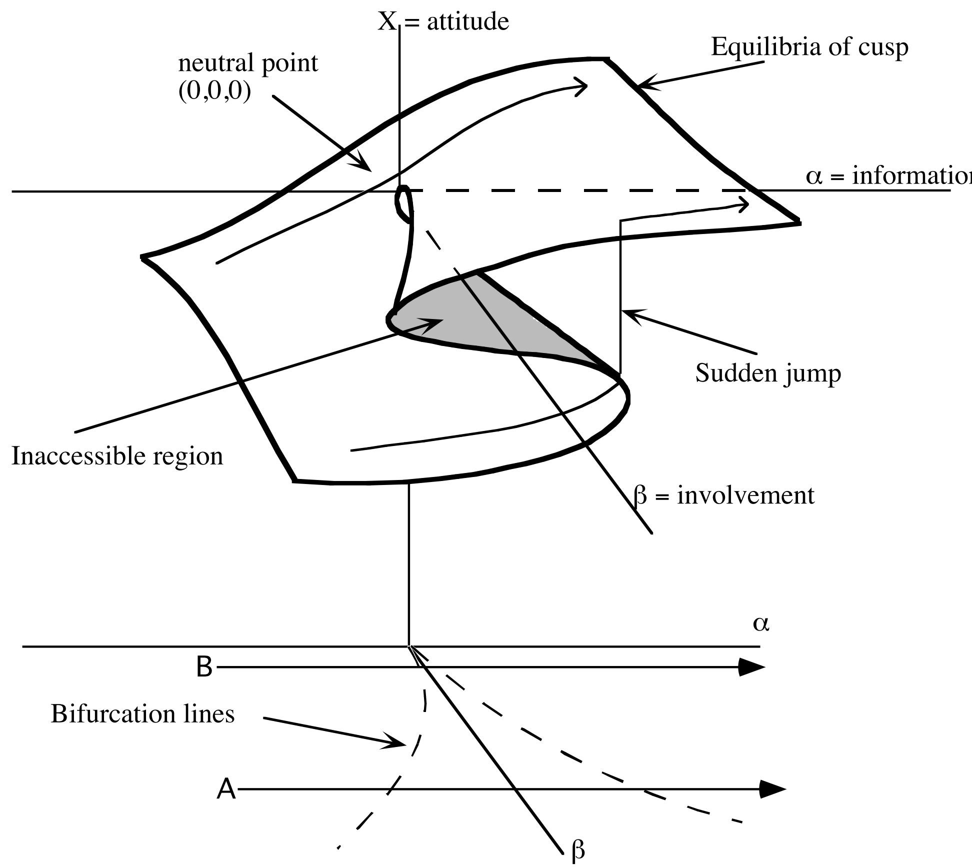

-2
β
AR
No motion
.75
1.5
0
0.1
bifurcation
lines
2
BRLC
4
data
points
Vert.
Motion
Hor.
Motion
0.9
6
-2
-1
0
1
2
α

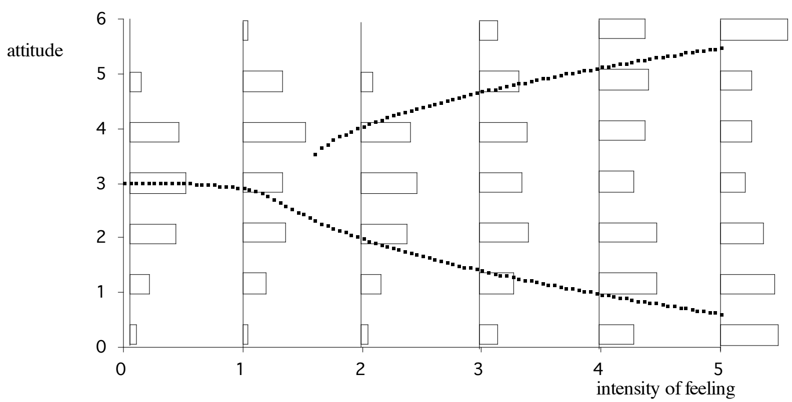
Attitudes

Sudden switches in attitudes:

Normal axis α:
Information (hysteresis)

Splitting axis β:
Involvement (divergence)



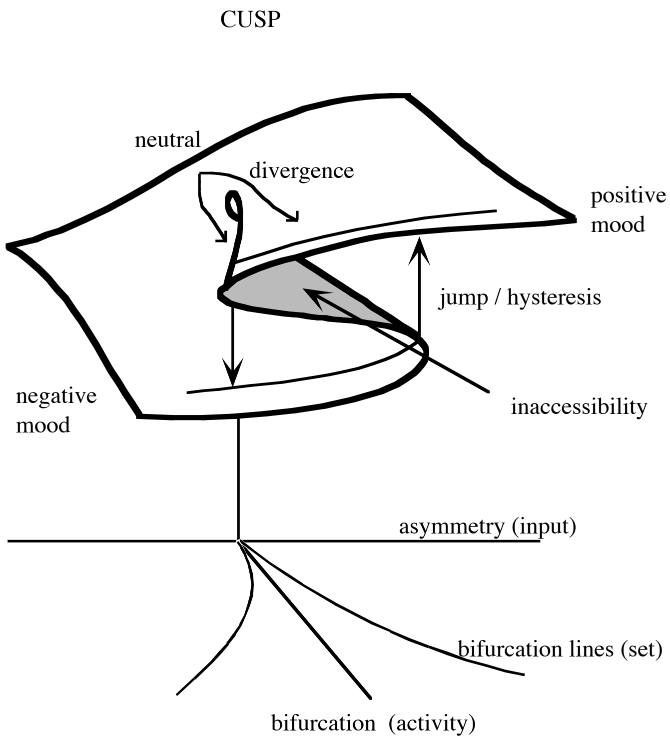


Np = number of positive attributes
Nn = number of negative attributes
α=Np-Nn
β=Np+Nn

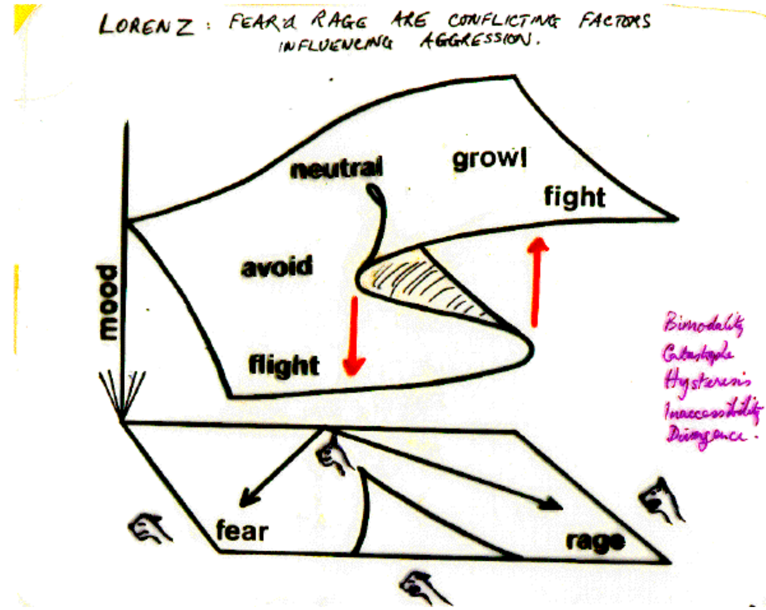
Ambivalence => $\alpha=0, \beta>>0$
Polarization => divergence

Emotions

A phase transition model for mother child interaction



positivity-approach negativity-avoidance transitions



In the main experiment, affect will be manipulated by means of affective stimuli (e.g. video films and music or computer tasks in which profit and loss are under strict experimenter control) and measured with facial expression, bodily motor behavior

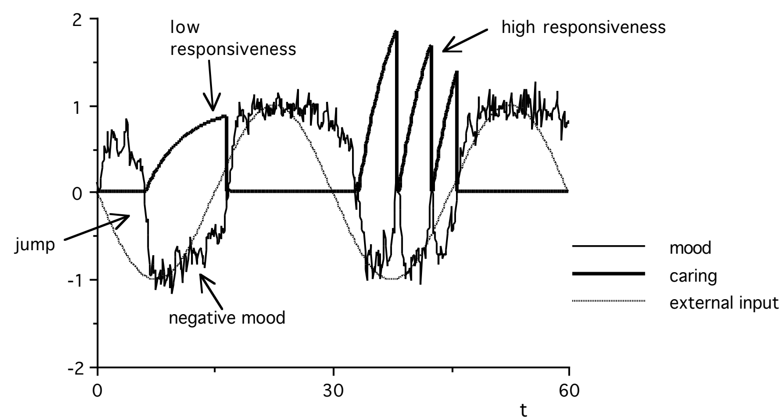
$dD/dt = -D_t + r_t D_t (1-D_t^2) - N_t = (r_t-1)D_t - r_t D_t^3 - N_t$

(equilibrium when $dD/dt = 0$)

$N_t = i E_t - s C_t$

$E_t = f(t)$: sinus ($v * t$) (wave model) or Poisson (λ) distributed bursts of size 1.

$dC/dt = u (1-C)$ if $D<0$, $C = 0$ if $D\geq 0$.



I&CD,2000

Try yourself

- Cusp model for
  - Falling in love
  - Quit smoking

Assignment

- option 1:
- Make your own cusp model for a psychological transition
  - choose behavioral and control variables
  - check flags to see whether model makes sense

- option:
- Build Zeeman Machine
- collect data
- fit in R with cusp.fit()

Zeeman machine

- http://lagrange.physics.drexel.edu/flash/zcm/